**From: Cees**

**To: Scott**

**Dd: 28-02-2025**

**Title suggestions**

These are just suggestions to stimulate further ideas. I am myself not yet fully convinced of any of them, let alone attached to any of them.

* *The problematic ‘elasticity’ of the Pearson correlation*
* *The upper and lower boundaries of Pearson’s r when applied to ordered-categorical data*
* *The uncalibrated correlation coefficient – or ‘how long is a piece of string?’*

**Introduction**

It is widely subscribed among quantitatively oriented social scientists that statistical tools that assume metric-level data should not be used in the analysis of ordered-categorical variables. [insert fn specifying ordered-categorical variables]. Doing so would endanger the validity of the results [fn: In the sense of Shadish, Cook and Campbell ‘s (2001) notion of statistical conclusion validity.] Yet, as we will demonstrate, such kinds of analysis are quite commonplace, particularly when using survey data which traditionally abound with ordered-categorical information.

This paper discusses one particular instance of such ‘inappropriate’ application of statistical procedures, namely the use of Pearson correlations. We do so not from an obsessive desire for methodological purity, because it is well known that many statistical procedures are quite robust for violations of their assumptions [fn: examples]. Instead, we do draw attention to a problematic consequence of this violation of assumptions that, as far as we know, has been rarely recognised in the literature. This is the problem of ‘elasticity’ of correlation coefficients between ordered-categorical variables. We describe below what we mean with elasticity, while sufficing at this place by noting that it results in incomparability of correlations between pairs of variables and a host of further problems downstream the analytical process.

This paper proceeds as follows ……

**Actual use of correlations based on ordered-categorical variables**

Before elaborating our argument about elasticity of correlation coefficients when their assumption of metric data is violated, we start by demonstrating that (and why) this is a commonplace practice indeed, and not a rarity that could well be ignored.

Correlation coefficients are mainly used for descriptive purposes about (linear) relationships between variables, or as a precursor for subsequent statistical procedures such as principal component analysis, factor analysis (exploratory or confirmatory), and so on. In the latter instance the correlations are often not explicitly reported but function as a kind of half-product to be used in further summarising analyses or in probes into latent structure.

A certain disdain among analysts for ‘merely’ descriptive analyses results in many recent scholarly articles not reporting matrices of bivariate correlations, but instead focussing on explanatory analyses. Nevertheless, a quick inventory of a single recent volume of four renowned academic journals yielded more than a dozen articles relying on Pearson correlations between ordered-categorical variables for descriptive purposes. [fn: These journals were EJPR vol 63, BJPS vol54, POQ vol??, and PSRM vol??] We do not include here references to these articles, as our aim is not to critique them, but rather to clarify that –even among manuscripts that survive the rigorous review processes of these highly ranked journals– Pearson correlations are quite frequently used for non-metric data. To make our statements here verifiable, we have informed the editors of this journal of the identity of these publications.

In addition to being used for descriptive analyses, correlations are frequently used in the context of analyses of the structure of the relationships between a set of variables. One can think of principal components analysis and various forms of factor analysis, but also of correspondence analysis, structural equation modelling, etc. Although full-information forms of such methods exist, their more common forms do not employ individual-level datapoints, but instead correlations between the variables of interest. These correlations are most often of the Pearson variety, which is the default in most of the available software. Applications of such analytical procedures on ordered-categorical variables are very numerous and, even if their reports do not present the actual correlations, they were nevertheless the indispensable basis upon which their published results rely. [add fn with a few counts of such publications in journals such as referred to above.]

In short, the use of Pearson product-moment correlations to characterise the relationships between non-metric variables is frequent and it is obviously regarded as acceptable practice in the research community. This in spite of the also widely shared warnings against such practice.

**Conditions contributing to the use of correlations for ordered-categorical variables**

One may wonder why the use of Pearson correlations on ordered-categorical variables is so widespread, and not rejected by colleagues, reviewers and editors. Some of the following factors may be relevant in this respect. [following still te be fleshed out as real text, it is now in memo-form for myself].

1. What else to do?
   1. Use Spearman rank-order correlations. Such correlation matrices are positive semi-definite. However, research questions about ranks are substantively different (and often less interesting) than questions about scores. Moreover, many researcher intuitively feel that they are discarding valuable empirical information when transforming ordered-categorical information into partial rank-orders.
   2. Use point-biserial, polychoric and polyserial correlations instead (describe these, and their intended use). However, matrices of such correlations are not guaranteed to be positive semi-definite, so that they could not be used as basis for PCA, FA, CA or SEM. Moreover, under some circumstances the procedures used to estimate these coefficients do not produce any outcome (cf Uebersax). Yet, some SEM software packages do provide the option of using these coefficients (most notably LISREL in combination with PRELIS) [note to self: check SPSS, Stata, SAS, MPlus etc]
2. Ordered-categorical variables are often referred to as ‘semi-interval’ (references). This is meant to indicate that their values may be regarded as close to equidistant, and the values hence as (close to) metric. There is often little evidence provided for such statements. When calibrating scores of ordered-categorical data against those produced by psychophysical measurements procedures (such as magnitude estimation) it is commonly found that the equidistant assumption is violated (Lodge 19??; others?), but not necessarily to a massive degree. As a result, correlations between ordered-categorical variables and their psychophysical counterparts are generally high (particularly when the tails of the distribution are sparsely populated). Therefore, the Pearson correlation appears to be quite robust for violations of the assumption of metric measurement and empirical researchers may be justified when choosing for the simplicity (and low cost) of standard practice versus methodological purity (and high cost) without much practical benefit.
3. Software functionality and defaults are strongly skewed towards the use of Pearson correlations. Default settings for PCA, FA and SEM procedures almost invariably specify Pearson correlations, while alternatives are not always available at all. [NB: PRELIS: # of categories > 10, use Pearson]
4. The practice of using Pearson correlations for ordered-categorical variables is so widely established that there seems to be no reason to doubt its validity. This is reinforced by examples in manuals and textbooks about statistical procedures [fn: provide examples].

For all these reasons, the use of Pearson correlations for ordered-categorical variables is rarely seen as problematic.

Yet, in spite of the apparent robustness of the correlation coefficient for violations of metric measurement assumptions, and in spite of the apparent acceptability of this usage, there is another problem that is rarely mentioned, namely the problem of the elasticity of correlations when using ordered-categorical variables. In the next section we specify what elasticity entails, and we describe some of its consequences for the validity of research findings.

**Elasticity of correlations between ordered-categorical variables**

* Introduce the problem and define elasticity in terms of the min/max of r
* Note that min/max is not the same for every pair of items
* Implies that r is not a common or shared and equally calibrated yardstick for assessing the strength of the linear relationship between ordered-categorical variables

**Minimum and maximum attainable values of the correlation coefficient**

* How to find the min/max
* Presentation of a software tool to calculate min/max

**Illustrations**

**Factors influencing the minimum and maximum of r**

* Analysis strategy
* Effect of differences in central tendency of the variables
* Effect of (differences in) their dispersion
* Effect of their skew and differences of their skew